MODEL PAPER

MATHEMATICS GRADE-12 (2nd Year)

BISE Examination 2027 — Smart Syllabus (ALP) — Paper Code: Math-XII

Time Allowed: 3 Hours **Total Marks:** 100

Instructions:

(1) Q. 1 (MCQs) is compulsory; each MCQ carries one mark. Cutting or overwriting an answer cancels it.

(2) Part-I: Attempt any **8** of 12 parts in Q.2, any **8** of 12 parts in Q.3 and any **9** of 13 parts in Q. 4. Each part carries 2 marks.

(3) Part-II: Attempt any **THREE** questions from Q. 5 to Q. 9. Each question has two parts i.e. (a) and (b) that carries 5 marks each.

(4) A non-programmable scientific calculator may be used. Take $g = 9.8$ m/s^2 wherever needed.

OBJECTIVE (MCQs) — Marks: 20

Q. 1 Choose the correct option. Each MCQ carries one mark. (1 × 20 = 20)

No.	Question	A	B	C	D
1	The y-intercept of the graph of $y = 2(x+1)(x-3)$ is:	6	-3	-6	3
2	If the graph of $y = f(x)$ is shifted 3 units upward, the new function is:	$f(x+3)$	$f(x-3)$	$3f(x)$	$f(x)+3$
3	If $y = \tan^{-1}(2x)$, then $\frac{dy}{dx} =$	$\frac{2}{1+4x^2}$	$\frac{1}{1+4x^2}$	$\frac{2}{1+2x^2}$	$\frac{2x}{1+4x^2}$
4	If $x = at^2$ and $y = 2at$, then $\frac{dy}{dx} =$	t	$2t$	$\frac{1}{t}$	$\frac{2}{t}$
5	The function $f(x) = x^3 - 3x$ has a relative maximum at $x =$	1	-1	0	3
6	$\int \cos 3x\, dx =$	$3\sin 3x + c$	$-1/3 \sin 3x + c$	$-3\sin 3x + c$	$1/3 \sin 3x + c$
7	$\int \frac{f'(x)}{f(x)}\, dx =$	$\ln\lvert f(x)\rvert + c$	$f(x) + c$	$\frac{[f(x)]^2}{2} + c$	$\frac{1}{f(x)} + c$
8	The area bounded by $y = x^2$, the x-axis and the lines $x = 0$, $x = 3$ is:	3	6	9	27
9	The differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ is:	separable	homogeneous	of order 2	of degree 2
10	The slope of the line $3x - 4y + 7 = 0$ is:	$\frac{3}{4}$	$-\frac{3}{4}$	$\frac{4}{3}$	$-\frac{4}{3}$

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No.	Question	A	B	C	D
11	The lines represented by $ax^2 + 2hxy + by^2 = 0$ are perpendicular if:	$a - b = 0$	$h^2 = ab$	$h = 0$	$a + b = 0$
12	The centre of the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ is:	$(-3,2)$	$(3,-2)$	$(6,-4)$	$(-6,4)$
13	The eccentricity of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is:	$\frac{4}{3}$	$\frac{3}{5}$	$\frac{5}{3}$	$\frac{5}{4}$
14	A car starts from rest with uniform acceleration 2 m/s^2. Its velocity after 5 s is:	2.5 m/s	7 m/s	25 m/s	10 m/s
15	If the position of a particle is $x(t) = t^3 - 2t$, its velocity at $t = 2$ s is:	10	4	12	6
16	By the Location of Root Theorem, a root of $x^3 - x - 4 = 0$ lies in:	$(0,1)$	$(1,2)$	$(2,3)$	$(3,4)$
17	The principal value of $\sin^{-1}(-1/2)$ is:	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$-\frac{\pi}{6}$	$\frac{11\pi}{6}$
18	The solutions of $\tan x = 1$ in $[0,2\pi]$ are:	$\frac{\pi}{4}$ only	$\frac{\pi}{4}, \frac{3\pi}{4}$	$\frac{3\pi}{4}, \frac{7\pi}{4}$	$\frac{\pi}{4}, \frac{5\pi}{4}$
19	If $\cos x = -1/2$ and x lies in the third quadrant, then $x =$	$\frac{4\pi}{3}$	$\frac{2\pi}{3}$	$\frac{5\pi}{3}$	$\frac{7\pi}{6}$
20	The domain of $\mathbf{r}(t) = \sqrt{t}\,\mathbf{i} + \ln(t-1)\,\mathbf{j}$ is:	$[0,\infty)$	$(1,\infty)$	$(0,\infty)$	$[1,\infty)$

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SUBJECTIVE — PART-I (Short Questions) — Marks: 50

Q. 2 Attempt any EIGHT parts. **(2 × 8 = 16)**

(i) Prove the following hyperbolic identity: $1 - \tanh^2 x = \operatorname{sech}^2 x$.

(ii) Given the function $f(x) = \sqrt{x}$, find the equation of the function after vertical stretch by factor 2.

(iii) A particle moving in a straight line starts from rest and is accelerated uniformly to attain a velocity of 30 km/h in 5 seconds. Find the distance travelled by the particle in the last three seconds.

(iv) The acceleration of a particle is $a(t) = \frac{1}{1+t}$ for $t \geq 0$. If the velocity is 0 m/s when $t = 0$, find the velocity at $t = 5$ seconds.

(v) Differentiate $y = x^2 \cos x$ with respect to the variable involved.

(vi) Find $f'(x)$ if $f(x) = \sin(2^x)$.

(vii) Find y_2 if $y = x^4 - 3x^2 + 4x - 5$.

(viii) Find the linear approximation of the function $f(x) = \sqrt{x^2 + 16}$ at $x = 3$.

(ix) Find the differential dy if $y = x^3 - 3\sqrt{x}$.

(x) Show that the relation $y = cx - 1$ is a solution of the differential equation $x\frac{dy}{dx} = 1 + y$.

(xi) Find the general solution of the differential equation $y\,dx + x\,dy = 0$.

(xii) A particle is moving in a straight line and its acceleration is given by $a = 2t - 7$. Find velocity v in terms of t if $v = 10$ m/sec, when $t = 0$.

Q. 3 Attempt any EIGHT parts. **(2 × 8 = 16)**

(i) Find the integral, $\int x^2 \left(x + \frac{1}{x}\right) dx$.

(ii) Find the anti-derivative $\int \frac{e^x}{1+e^{2x}}\, dx$.

(iii) Evaluate $\int \ln x\, dx$.

(iv) Solve by partial fraction $\int \frac{5x+7}{(x+1)(x+2)}\, dx$.

(v) Evaluate the definite integral $\int_1^2 (x^2 + x)\, dx$.

(vi) For what value of k if the lines $x + y - 4 = 0$, $2x - y + 1 = 0$ and $kx + 2y - 10 = 0$ are concurrent.

(vii) Find the area of the triangle formed by the points $A(-3,2)$, $B(4,-1)$, $C(1,5)$.

(viii) Workout the value of k such that the lines represented by the equation $(k-8)x^2 - 9xy + 3ky^2 = 0$ are perpendicular to each other.

(ix) Find an equation of the circle with centre at $(5,7)$ and radius 2.

(x) Find the equations of tangents to the circle $x^2 + y^2 = 4$ which are parallel to the line $3x + 4y + 1 = 0$.

(xi) Write the equation of the parabola with focus $(-3,2)$ and directrix $x = 5$.

(xii) Find the length of the latus rectum of the hyperbola $9x^2 - 16y^2 = 144$.

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Q. 4 Attempt any NINE parts. **(2 × 9 = 18)**

(i) Find the real root of $f(x) = x^3 - x - 1$ correct to two decimal places using the graphical method, by finding some interval of sign change.

(ii) Find the real root of $f(x) = x^3 - 3x - 10$ correct to 3 decimal places using the bisection method, within the interval of sign change [2,3].

(iii) Use Trapezoidal Rule to approximate the value of $\int_0^4 \sqrt{x}\ dx$ taking $n = 4$.

(iv) Find the principal value of $\tan^{-1}(-\sqrt{3})$.

(v) Find the domain of $f(x) = \sin^{-1}(5x - 3)$.

(vi) Prove that $\cos(\sin^{-1}x) = \sqrt{1 - x^2}$.

(vii) Solve the trigonometric equation $\sin x = \frac{1}{\sqrt{2}}$.

(viii) Solve the trigonometric equation $\sqrt{2}\cos x \sin x - \sin x = 0$.

(ix) A person standing 20 meters away from the base of a tree measures the angle of elevation to the top of the tree as 30°. Find the height of the tree.

(x) Give two real-life examples where vector-valued functions are necessary.

(xi) Given $x(t) = t^2 + 1$, $y(t) = \sqrt{t}$, construct $\mathbf{r}(t)$.

(xii) Find the first-order derivative of $\mathbf{r}(t) = (t^2 + 1)\hat{\imath} + (t^3 + t^2 + 3t)\hat{\jmath}$ with respect to t.

(xiii) Find the first-order derivative of $\mathbf{r}(t) = te^{-t}\,\hat{\imath} + \cos(3t)\,\hat{\jmath} + \ln(1 + t^2)\hat{k}$ with respect to t at $t = 0$.

SUBJECTIVE — PART-II (Long Questions) — Marks: 30

Attempt any THREE questions. Each part carries 5 marks. **[3 × (5 + 5) = 30]**

Q. 5 (a) Solve the equation: $3^{(x+1)} - 2 \cdot 9^x + 9 = 0$.

(b) Prove that: $\sin^{-1}\frac{12}{13} - \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{33}{65}$.

Q. 6 (a) Using the Second Derivative Test, find local extreme values of $f(x) = x^4 - 8x^2 + 2$.

(b) Solve the initial value problem: $\frac{dy}{dx} - x = xy^2;\ y(0) = 1$.

Q. 7 (a) Find two positive integers whose sum is 12 and the product of one with the square of the other will be maximum.

(b) The vertices of $\triangle PQR$ are $P(1,2)$, $Q(5,-6)$ and $R(7,-2)$. Find the equations of all three medians.

Q. 8 (a) Find the area bounded by the parabola $y = x^2 - 6x + 8$, $x = 2$, $x = 4$ and the x-axis.

(b) A particle is projected vertically upwards with a velocity 100 m/s, and another is let fall from a height of 200 m at the same time. Find the height of the point where they meet each other.

Q. 9 (a) Find the equation of the ellipse as the locus of points $P(x, y)$ such that the sum of the distances from P to the points $(-3,0)$ and $(3,0)$ is 10.

(b) Find the real root of the nonlinear equation $e^x = 4x$ near $x = 2$, accurate to 3 decimal places, using the Newton–Raphson method.

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